

Title

Proportional Reasoning (Grades 6-8)

Target Audience

This course is intended for pre-service and in-service grades 6-8 teachers.

Course Description

This course is designed to give middle school educators new and effective strategies for teaching ratio, proportion, and percents, as well as ways to further students' proportional reasoning skills. Learners will explore the ways in which students approach proportion problems, and collaborate with other learners to find ways to target student misconceptions about proportionality. Learners examine different methods of comparison, various ways to solve proportion problems, and intuitive approaches that students might use. As a final task, learners practice implementing their newly learned teaching methods and tools into the curriculum by crafting a lesson plan designed to teach a concept of proportion.

Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Learners will gain:

- A deeper understanding of proportionality.
- Different strategies for how one might teach ratio, proportion, and percents.
- An appreciation for and a deeper understanding of why this material is so complex for students.

Outline of Content and Assignments

After previewing the course introductory information, learners will proceed to the Assignments section to complete the following six sessions (or parts), working through each in order. In these sessions, they will solve mathematical problems as a learner, and then use that experience to reflect on the misconceptions these problems generate in students and discuss strategies for helping students overcome their misconceptions. Learners will also read NCTM's *Principles and Standards for School Mathematics*, as well as solve sample problems that have appeared on state tests, so that they can correlate the mathematics they've explored in this course to the mathematics students are required to learn. As a final task, learners will create a portion of a lesson plan to follow one of the activities in this course, coinciding with the teaching and learning of function and its importance to the mathematical education of high school students. The sessions are as follows:

Session 1: Orientation

Session 2: Making Comparisons: Exploring Proportionality

Session 3: Absolute vs. Relative Comparisons

Session 4: Understanding Students' Thinking



Session 5: Teaching Proportions
 Session 6: Pulling It All Together

Session 1: Orientation

Learners will:

- Reflect on expectations and prior knowledge
- Read about research on teaching proportional reasoning
- Introduce themselves to other participants

Read

- An article that will familiarize learners with the most current research about the content and pedagogy related to the mathematics of proportional reasoning.
- A case study of a teacher’s trials and errors with lessons

Watch

- A short, informative video about preparing themselves to think like a learner as well as a teacher for this course

Explore

- Reasoning problems on a State Assessment

Respond in an Online Journal

- Expectations for the course
- Prior knowledge of proportional reasoning

Participate in an Online Discussion

- Introduce themselves on the discussion board.
- Exchange thoughts and ideas regarding the case study of the teacher’s lessons.

Session 2: Making Comparisons: Exploring Proportionality

Learners will:

- Understand several different ways to think about comparing two or more ratios.
- Understand the nature of a common misconception related to the use of addition versus multiplication in proportional relationships.
- Discuss some ways to determine whether students understand proportional relationships.

Read

- A report in *The Mathematical Education of Teachers*, published by the Conference Board of the Mathematical Sciences. The section on “Teaching Mathematical Reasoning in the Middle Grades” highlights some current directions in teaching proportional reasoning. It is suggested that learners do this reading before beginning the activities.
- “Math Background.” This short piece from the *MathScape Buyer Beware* unit provides some background information on the basics of ratios and proportions.

Explore

- The “Orange Juice Mixtures” problem. This first problem, often cited in research on proportional reasoning, addresses one of the deeply held misconceptions about proportionality. As learners work on the problem, they should think about how multiplication is important in the problem. Print out the “Comparing Mixtures” worksheet and use the “Mixing Orange Juice” interactive to check answers.

- The “Sorting Rectangles” problem. Proportional reasoning appears in a number of different content areas within mathematics. This problem looks at proportionality in geometry. Again, the emphasis in the problem is on developing your understanding of how proportional reasoning relates to multiplicative relationships. Note: This problem is worded for students who have not yet learned about similarity. It is intended as a first introduction to proportional reasoning in similarity.

Participate in an online discussion

- Respond to the “Orange Juice Mixtures” problem or the “Sorting Rectangles” Be sure to explain how you know these questions will review their mathematical understanding.

Complete the Assignment

- Consider the following situation. One classroom in a school has 15 girls and 12 boys. An administrator wants to maintain the same ratio of girls to boys in a second classroom. To do so, she creates a class with three fewer girls and three fewer boys for a total of 12 girls and 9 boys.
 - Explain what's wrong with her reasoning.
 - Suggest another class composition of any total number of students that has the original ratio of girls to boys and explain how you decided it was correct.

Session 3: Absolute vs. Relative Comparisons

Learners will:

- Recognize the differences between absolute versus relative comparisons and how a lack of understanding of these differences is the source of many misconceptions for math students.
- Identify examples of absolute versus relative comparisons in a variety of proportional-reasoning problems.
- Describe the relevance of understanding absolute and relative comparisons in some real-life situations.

Read

- “Proportional Reasoning” in Chapter 7: Proficiency with Numbers in the book, *Adding It Up* (pages 241-244). *Adding It Up* has received a lot of attention as the result of a study done by the National Research Council (the research arm of the National Academy of Sciences) about the important mathematics, particularly in the number strand, students should learn and the related teaching issues. These pages describe several aspects of proportional reasoning that are key for students to learn. For your own interest, you may want to look at the Executive Summary for the report, which describes the rationale for the report.

Explore

- The “Restaurant Problem”
- Complete the “Who's in the House?” problem. Use the tools provided.

Complete the Assignment

- From the “Who's in the House” problem, write two statements that describe an absolute comparison that you can make from this data and two statements that describe a relative comparison that you can make from this data. For each statement, describe who might want to use or know this.

Participate in an online discussion

- How might deepening a student's understanding of relative and absolute comparisons help to overcome this misconception?

Session 4: Understanding Students' Thinking

Learners will:

- Identify and describe areas of frequent student misconceptions in proportional reasoning.
- Examine students' work in which they try to reason proportionally in a number of different middle school mathematics areas.
- Become familiar with the technique of conducting student interviews.

Read

- "PSSM Connections Standard for the Middle Grades." This document describes the expectations, gives examples for connecting and integrating related mathematics topics, and includes a useful proportional-reasoning example.
- Read the article, "Lessons from an Interview,"

Explore

- Complete the "Different Ages" problem.
- Complete the "Marigolds" problem.
- Print out the "Student Misconceptions" activity.

Watch

- Watch the "Conducting Student Interviews" video in which the interviewer describes what she is going to try to do with the students.
- Watch the "Which Grew More?" video, in which several students describe their solutions to the "Marigolds Problem." Note where their misconceptions lie with this problem.
- Watch the "George's and Brian's Thinking" video, which contrasts two students' solutions to the problem of Sean's and Emily's test scores.
- The next three video clips capture three different students' thinking about the "Different Ages Problem." As you watch the videos, note how the interviewer tries to follow each student's reasoning even when it is flawed.
 - Watch the "Ashlee's Pictures" video
 - Watch the "Juan's Calculations" video
 - Watch the "Brian's Pairs" video

Participate in an online discussion

- Describe one piece of information in the video that the teacher may not have gotten from using other assessment techniques or tools. Do you think this process of student interviewing is worthwhile? Why or why not?

Session 5: Teaching Proportions

Learners will:

- Understand and be able to explain a variety of methods for solving proportion problems.
- Analyze student work to uncover the students' thinking and explain the students' probable reasoning.
- Understand the mathematical basis for cross-multiplication.

Read

- "Why Does Cross-Multiplication Work?" Have you ever wondered why you can cross-multiply to solve proportions? This brief piece provides an explanation for the mathematics that results in the cross-multiplication algorithm.
- An article from the *Mathematics Teacher* (NCTM's high school journal) gives a comprehensive explanation of the various aspects of proportional reasoning that students need to master.
- "A Proportion Puzzle." In this case, a teacher tries to teach students to solve proportions using cross-multiplication. At the end of the lesson, several students come up with three other

methods for solving the problem that they claim are more intuitive. The teacher wonders in what direction to go next. This case will provide some food for thought for the discussion that follows.

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Explore

- Print out and solve the “Salt Water Taffy Problem” by using two different solution methods.
- Print out “Four Solutions from Students” to see four different ways that students solved this problem. Try to figure out what each student did to find the missing values.

Participate in an online discussion

- Some educators believe that teaching algorithms without a mathematical understanding behind them is meaningless. Others believe that students need to be given an algorithm. However, students sometimes express frustration when they have their own solution methods that work but differ from the teacher’s prescribed method. How would you reconcile these important concerns? Support your comments with mathematical examples.

Complete the Assignment

- Describe what each student did in his or her solution in “Four Solutions from Students.” Be sure to include a step-by-step description of the student’s method and an explanation of why it works.

Session 6: Pulling It All Together

Learners will:

- Create a lesson plan based on the content and pedagogy learned in this course.
- Consider alternate teaching strategies that might be employed in teaching percents and proportions.

Complete a final project

- Create a lesson plan involving proportional reasoning that you or another teacher might use in the classroom. The lesson plan should:
 - Outline clear mathematical goals
 - Describe what prerequisites, if any, students need to fulfill before the lesson
 - Define what happens in the beginning, middle, and end of the lesson
 - Include one or two engaging activities
 - Highlight the teaching strategies to help students achieve the goals
 - Identify the common student misconceptions related to the topic and the strategies for addressing them
 - Include some form of wrap-up (discussion questions or culminating writing task)
 - Give guidance for the teacher on what topics should come out of the lesson
- The project should be approximately three pages in length.
- The final project may include activities that the learner has done in this course. However, the project should build on the given activities or extend them in some way. The project should also include original activities or activities from a source other than this course.

Respond in the Online Journal

- Reflections on the learning from this course, and what one or two ideas do you take away from this course that you can use?
- How will you continue your professional development?

Schedule

This course is scheduled to take approximately 15 hours to complete readings, activities, video, assignments, reflections and a final project.

Requirements

Learners are expected to:

- Complete all assignments
- Maintain an online journal
- Participate regularly in discussion boards

Evaluation

This course is evaluated on a letter grade basis, and may be available for graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

Materials (hardware, software, plug-ins)

Technical Requirements

- Word processor
- Internet service provider
- E-mail

Academic Dishonesty Policy

To be inserted by university institution only