Seeing Math Course Syllabus

Title

Seeing Math™: Systems of Linear Equations

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

Learners taking this course should:

- Have familiarity with different symbolic forms for expressing a linear function (equation) (standard form, point-slope form, slope-intercept form);
- Have familiarity with methods for solving a system of linear equations (graphing, substitution, and elimination – also known as linear combination);
- Have familiarity with the rules of equality that apply to solving equations;
- Be able to recognize that changes in the symbolic form of a line affect the graphic representation and vice versa;
- Be ability to graph multiple functions on the same axes.

Note: this course does note address matrix methods for solving systems of linear equations.

Course Description

What does it mean to solve a system of linear equations and what do all the procedures used to solve systems of linear equations have in common? The Systems of Linear Equations course is designed to help learners and ultimately their students make the connections between the symbolic, graphic, and tabular representations of systems of linear equations. It further provides learners with an understanding of each step in the solution process to help them master the skills needed to solve systems of linear equations.

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Learners will develop a deeper understanding of what it means to find the solution to a system of linear equations.

Learners will be able to:

- Distinguish solutions to systems of linear equations from solutions to individual equations.
- Represent the solution to a system of linear equations symbolically, graphically, and in tables.
- Use multiple representations to interpret and understand different solution sets—one solution, no solution, or an infinite number of solutions.

Learners will recognize how valid procedures for solving systems of linear equations relate to the solution set.

Learners will be able to:

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- Predict the symbolic, graphic, and tabular results of intermediate steps in the solution process.
- Demonstrate, in multiple representations, how valid operations may alter the solution set for individual equations in the system while preserving the solution set of the system.

Learners will recognize the steps students take when solving systems of linear equations.

Learners will be able to:

- Interpret student thinking using multiple representations of systems of linear equations
- Identify when students perform the procedures for solving a system of linear equations without fully understanding their meaning
- Link their curricula and teaching practice to strategies that support student learning of concepts that are fundamental to understanding systems of linear equations

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal; the discussions in the discussion forum are designed to allow learners to glean information from other learners' experiences.

This five-week course is taken entirely over the Internet. Learners should expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course, learners focus on creating and completing a final project.

Learners also come away with a tangible benefit—interactive software and activities to use with students. These tools are used within the course, such that learners are thoroughly familiar with them. In addition, learners are provided with alternative activities that do not require computers if computer resources are not available for classroom use.

Week 1: Orientation

Much of the *Orientation* session is spent getting to know the courseware and meeting colleagues online. Learners also read about our approach to learning and teaching data analysis, look at the NCTM Standards as well as their own state standards, and begin entering notes and reflections in their course journals.

Learners will:

Read:

- *The Landscape of Learning* – A discussion on the underlying principles behind the learning in this course and what they should expect.
- *The Landscape of Systems* – A discussion about the importance of students learning a solution strategy for solving systems of linear equations.
- National Council of Teachers of Mathematics *Algebra Standards*.
- Local state algebra standards.

Write in journal:

- Reflect on insights and ideas gained from the *Landscapes on Learning* reading.
- Reflect on insights and ideas gained from the *Landscapes on Systems* reading.

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- Reflect on insights and ideas related to the national standards and how their curriculum currently addresses them (or does it)?

Participate in online discussions:

- Post messages in the Discussion Board, Week 1, *What's in a Name?* forum by introducing yourself and describing qualities about yourself using the letters in your first or last name. Read what colleagues have to say and respond to at least two other posts.
- Post messages in the Discussion Board, Week 1, *Speaking of Systems* by sharing some of their thoughts on *the two* Landscapes readings—of learning and of systems. Read what colleagues have to say and respond to at least two other posts.

Week 2: Math Focus

The focus of this session is to examine problem-solving strategies. A better understanding of the graphic and tabular as well as symbolic representations of problem solving steps can help students develop better solution strategies, and a deeper understanding of what it means to solve a system of linear equations.

Learners use the System Solver online interactive tool to solve systems of linear equations and observe in multiple representations the changes that take place as you complete each step.

Learners will:

Read:

- *Snapshots from the Curriculum*: Standard way curricula approach solving systems of equations, and its limitations.
- *Observing Your Processes*: Description of what to observe as learners complete the activities.

Complete activities and assignments:

- Diving In: Work on challenges using both paper and pencil by exploring the steps involved in solving systems of linear equations. Make observations about the effects of those operations on the solution sets of the systems.
- System Solver Warm-up: Warm-up activity for learners to acquaint themselves with a new software tool before using it with an activity
- Diving In: Checking Your Predictions – Use the System Solver to solve the Diving In problems, so that they can experience how technology can provide insight into what it means to solve a system of linear equations.

Write in journal (not required):

- Reflect on approaches students used to solve the *Diving In: Checking Predictions* challenges by looking over their steps and predictions for each challenge.
  - Do they see a pattern in their predictions?
  - Were they more comfortable with one technique over another?
  - Did Jackie, Maya, and Shane start solving their problem the way they would have?
  - How might they have begun or continued differently had they not been directed to continue the process they'd already started?
  - How did it feel to experience the step-by-step process in this way?
  - What were some of the ideas that led you to your predictions? Why?
- Reflect on the challenges in the *Diving In: Making Predictions* activity after using the System Solver. Learners consider these questions:

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- How accurate were your predictions?
- How did the System Solver help them analyze and gain new insight into the graphic and tabular results of each step of the solution process?
- Were there any surprises?
- Where did they get stuck?
- What questions remain unresolved for them?

Participate in online discussions:

- Post summary of the Diving In reflection questions to the Discussion Board, Week 2, *Diving In: Making Predictions.* Read what colleagues have to say and respond to at least two other posts.
- Post summary of responses to Discussion Board, Week 2, *Diving In: Checking Your Predictions.* Read what colleagues have to say and respond to at least two other posts.

Week 3: Student Thinking

The focus of this session is student thinking. In this session, learners examine student work and read excerpts from class discussions that took place during a problem solving session. What do students really understand, and how do they get there? To help, learners watch videos of mathematics specialist, Diane Briars, Ph.D., comment on aspects of student learning. Afterwards, they analyze some unexpected solutions, with and without the System Solver.
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Learners will:

Read:

- *Meet the Students* –Background information about the students whose work will be analyzed.
- *Tracking Student Thinking Part 1* – Look at student work and read transcripts of students' discussions as they solve the Diving In problems.
- *Tracking Student Thinking Part 2* – Look at student work and read transcripts of students' discussions as they solve the Diving In problems.

View videos:

- Mathematics specialist, Diane Briars, discussing student thinking,

Complete activities and assignments:

- Diving Deeper: A Closer Look at Solutions: Analyze systems of linear equations that yield unexpected results. Solve systems of linear equations—first with paper and pencil and then with the System Solver.

Write in journal (not required):

- Reflect on issues and insights from Briar's commentaries considering the following:
  - Students may have different notions of equivalence. What does Briars infer about student thinking?
  - How do the ways students express themselves influence our understanding of what they are thinking?
  - What are some ways to probe further?
  - What observations have learners made of their own students' interpretations of the mystery behind procedures for solving systems of linear equations?
  - What recommendations does Briars make and what strategies do learners find useful when using technology to unravel the mystery and build understanding?
  - How have learners' students applied concepts they've previously learned to understanding new concepts or contexts?

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- Reflect on how their students would approach solving a system of linear equations considering the following:
  - What would your students do upon reaching the "correct" result?
  - How would you help them to make sense of their answer?
  - How can legitimate operations give both true and false results?
  - How would you explain this use of the = sign?

Participate in online discussions:

- Review notes of what they considered interesting or significant in the student work and discussion excerpts from the two *Tracking Student Thinking* activities. Summarize two or three issues or ideas that stood out and post summaries to the Discussion Board, Week 3, *Tracking Student Thinking.* Read what colleagues have to say and respond to at least two other posts.
- Review notes, and post comments on the Discussion Board under Week 3, *Specialist Commentary* on what they considered as significant from the commentaries of Diane Briars? What insights did Dr. Briar's commentaries spark? Read what colleagues have to say and respond to at least two other posts.

Week 4: Your Classroom

In this session, learners try an unusual activity—"unsolving" a problem by starting with a solution set and working backwards to create a system of linear equations. This activity reflects issues Diane Briars addresses in her final commentary in which she contrasts traditional textbook approaches to solving systems of equations with the System Solver's multiple representations. Afterwards, learners compare their curriculum to the approach taken in this course. Activities that can used be with students will be provided.

Learners will:

Watch a video:

- Mathematics specialist, Diane Briars, contrasting traditional textbook approaches to solving systems of equations with the System Solver's approach.

Complete activities and assignments:

- *Unsolving a Problem*: Start with the solution to a problem and work backwards to create a new system of linear equations using the System Solver.
- *Picture an Activity*: Using one problem from their textbooks and at least two strategies to guide the activity (either from the list provided in the course or from their own ideas), describe how they would construct an experience that engages their students in exploring systems from the perspectives used in this course.
- Generalizing about Systems: Activity that provides a series of questions about characteristics that can apply to any system of equations.

Write in journal (not required):

- Reflect on issues and insights from Briar's commentaries considering the following:
  - How does Briars describe the advantage of starting from the solution and working backwards as a way to develop student understanding?
  - How would you incorporate multiple representations into this strategy?
- Reflect on the Unsolving a System activity and consider the following:
  - Under normal circumstances, the only time one would add two equations to each other would be to eliminate a variable. Here, it is likely that several linear combinations were performed. What are the similarities and differences between a linear combination to eliminate a variable and a linear combination in which variables are *not* eliminated?

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- Did this activity support the generalizations made based on their experiences in the Diving In activities? How?
- Could linear combinations have been selected that would lead the graph of either equation to have a specific slope and/or to pass through a specific point? If not, why not?

Participate in online discussions:

- Review notes and write summaries of what was significant in the *Unsolving a Problem* activity and reflections." Post summaries on the Discussion Board, Week 4, *Unsolving a Problem.* Address the connections between their own experiences and with the commentary by Diane Briars. Read what colleagues have to say and respond to at least two other posts.
- Post descriptions created in *Picture an Activity* in the Discussion Board, Week 4, *Links to the Curriculum.* Read what colleagues have to say and respond to at least two other posts.

Week 5: Your Plan

In this session learners look back over the landscape of ideas they explored during this course. They review their thoughts and records to consolidate their learning experiences and build upon the work completed over the past weeks. Learners create a final project designed to integrate the mathematical concepts developed throughout the course into their instructional program. Learners also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- A review of the major topics addressed in this course

Complete activities and assignments:

- Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy that the learner wants to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills.
- Post in the Discussion Board, Week 5, *Gallery of Plans-learners' completed plans.*

Participate in an online discussion:

- Post their Aha!, Oops! and/or Whew! comments on the Discussion Board Week 5, *Aha! Oops! Whew!* Say goodbye to fellow learners, share their memories of this course experience, and plan for continued professional contact. Read what colleagues have to say and respond to at least two other posts.

Write in journal (not required):

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Schedule

This course is scheduled for 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners focus on creating and completing a final project.

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Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System
For the best experience, use the newer operating systems: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser
Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.5 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course is evaluated on a letter grade basis, and may be available for graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

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Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require learners to share their thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criterion for each assignment. The facilitator will apply individual criteria as necessary (for

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instance, not all activities require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.

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Action Plan Rubric

Criteria	**Does not meet expectations**	**Meets expectations**	**Exceeds expectations**
Completion of Assignment and Timeliness	Assignment is not completed and/or submitted on time.	Assignment is completed and submitted on time.	N/A
Description of the instructional strategy	A specific action or instructional strategy is not included and/or reason(s) why this strategy was selected are not included.	A specific action or instructional strategy is included, along with the reason(s) why this strategy was selected.	A specific action or instructional strategy and reason(s) why this strategy was selected are included. In addition, specific examples from experiences in the course or from personal reflections that were instrumental in the selection of the stated strategy are included.
Goals	The goals for the teacher and/or the students based on the instructional strategy selected are not included.	The goals for both teacher and student based on the instructional strategy selected are included.	The goals for both teacher and student based on the instructional strategy selected are included. Goals are SMART (see note below).
Objectives	The specific performance objectives teacher will do to accomplish the stated goals for the selected strategy are absent.	The specific performance objectives the teacher will do to accomplish the stated goals for the selected strategy are clearly described.	The specific performance objectives the teacher will do to accomplish the stated goals for the selected strategy are clearly described. A rationale for each of the stated objectives is provided.
Potential activities	The plan does not delineate and/or describe the specific activities/tasks to be undertaken to meet the objectives.	The plan delineates and briefly describes the specific activities/tasks to be undertaken to meet the objectives.	The plan delineates and briefly describes the specific activities/tasks to be undertaken to meet the objectives. The activities indicate attention to ensuring the needs of all students are met.

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Relevance	How the scope of the plan is relevant to the study of algebra, or to the students, or the school or district, and to the quality of educational practice is not stated.	How the scope of the plan is relevant to the study of algebra, or to the students, the school or district goals, and to the quality of educational practice is stated.	How the scope of the plan is relevant to the study of algebra, to the students, the school or district goals, and to the quality of educational practice is stated.
Evaluation	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are not described.	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are described.	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are described. A brief student evaluation survey is also included.
Portfolio	Pieces of evidence from the proposed activities that will be collected for each objective are not identified.	Pieces of evidence from the proposed activities that will be collected to support each objective are identified.	Pieces of evidence from the proposed activities that will be collected to support each objective are identified. Also included is a rationale statement for selecting at least four pieces of evidence.

Note: SMART refers to Specific, Measurable, Attainable, Relevant, Time-bound.

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Lesson Plan Rubric

Criteria	Does not meet expectations	Meets expectations	Exceeds expectations
Completion of Assignment and Timeliness	Assignment is not completed and/or submitted on time.	Assignment is completed and submitted on time.	N/A
Goals	Goals are not provided.	Goals clearly state the purpose of the lesson. There is evidence of the alignment of the goals, objectives, learning activities, and assessment methods described in the lesson plan.	N/A
Objectives	Objectives are not provided.	Objectives are performance-based, state what students should know and be able to do as a result of learning instruction. Objectives are appropriate for the grade level and students identified. There is clear evidence of the alignment between the goals, objectives, learning activities, and assessment methods described in the lesson plan.	N/A
Standards Addressed	National and state content and technology standards are not addressed.	National and state content and technology standards are listed.	N/A

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Prerequisites	Prerequisite knowledge and skills are not provided, or prerequisites are vague, or prerequisites are not appropriate.	Appropriate prerequisite knowledge and skills needed by students are provided.	Appropriate prerequisite knowledge and skills needed by students are provided. An explanation of their importance to the learning is provided.
Materials	Materials and resources are not listed, or only a partial list is provided.	A complete list of materials, resources, and detailed descriptions of any special considerations and/or advanced preparations are provided.	A complete list of materials, resources, and detailed descriptions of any special considerations and/or advanced preparations are provided. A list of additional/alternative materials and resources is also provided.
Lesson Overview	The lesson overview is not provided or lesson overview is incomplete, vague or unclear.	The lesson overview provides a brief statement that summarizes key aspects of the lesson.	N/A
Teaching Strategy	Lesson design does not document clearly the teaching strategy that needs to be implemented, whether it is direct instruction or learner-centered instruction.	Lesson plan documents clearly the appropriate teaching strategy needed for the lesson, whether it is direct instruction or learner-centered instruction.	Lesson plan documents clearly the appropriate teaching strategy needed for the lesson, whether it is direct instruction or learner-centered instruction. A rationale for the selection based on course readings and best practices is provided.

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Lesson Procedures		Lesson procedures provide a detailed, step-by-step description of the lesson. They include: Introduction — how students will be introduced to the goals and what is expected of them Main activity — how the teacher will facilitate the learning experience Conclusion — how the teacher will bring closure for students and provide feedback Extension — how the lesson will be extended.	Lesson procedures provide a detailed, step-by-step description of the lesson. They include: Introduction — how students will be introduced to the goals and what is expected of them Main activity — how the teacher will facilitate the learning experience Conclusion — how the teacher will bring closure for students and provide feedback Extension — how the lesson will be extended. Procedures include strategies for differentiated instruction.
Assessment	Assessment is not provided, is incomplete, and/or vague. There is not a clear relationship between the assessment and the skills taught during the lesson.	Assessment(s) to be used to evaluate students' learning is (are) provided. There is a clear relationship between the assessment(s), the content, and the skills taught during the lesson.	Assessment(s) to be used to evaluate students' learning is (are) provided. There is a clear relationship between the assessment(s), the content, and the skills taught during the lesson. Assessment(s) incorporate(s) a consideration for diverse student needs. A rationale for the selection of the assessment technique(s) based on course readings and best practices is provided.

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