Title

Seeing Math: Quadratic Functions

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

Learners taking this course should have:

- Good grounding in basic Algebra and comfort with linear functions, slope, and graphing straight lines
- Familiarity with the various forms that a quadratic function may take: polynomial $y = ax^2 + bx + c$, vertex $y = a(x-h)^2 + k$, and root $y = a(x-r_1)(x-r_2)$
- Familiarity with the symmetry of the parabola and with transformations of a parabola

Course Description

By modeling and problem solving, learners identify and describe the characteristics of quadratic functions. As they solve a puzzle, learners use multiple representations—tables, graphs, and symbolic expressions—as powerful tools to model physical situations and predict patterns. Then they connect the multiple representations back to the original referents in the puzzle. Through problem solving, observation of students, discussion, and readings, learners experience the value of varied solutions to a problem, develop strategies for working with symbolic manipulations, distinguish local and global patterns, and examine ways to support student understanding. Learners adapt curriculum materials to enrich their teaching of quadratic functions and equations, and also come away with a tangible benefit—interactive software and activities to use with students (along with alternative activities for classrooms that lack computer resources.)

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Search for patterns and use quadratic functions to model physical situations. Learners will be able to:

- Engage in problem solving and understand the value of different solutions to the same problem
- Describe a physical situation using symbols, and then interpret the symbols by mapping them back onto the physical situation
- Distinguish between *recursive* and *explicit* patterns

Interpret the meaning and characteristics of quadratic functions as they appear in different representations. Learners will be able to:

- Interchange symbolic, graphic, numeric, and verbal representations of the same quadratic function with versatility
- Link characteristics in one representation to corresponding characteristics in another representation
- Use appropriate terminology to describe various characteristics

Link a personal understanding of quadratic functions to their curriculum and to students' understanding. Learners will be able to:

- Gain skill in listening and interpreting student thinking accurately
- Explore and generate ways to support students' understanding
- Relate their curriculum's treatment of quadratic equations to a thorough understanding of quadratic functions

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners will proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal; the discussions in the discussion forum are designed to allow learners to glean information from other learners' experiences.

This five-week course is taken entirely over the Internet. Learners should expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners will complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Learners will also come away with a tangible benefit—interactive software and activities to use with students. These tools will be used within the course, such that learners are thoroughly familiar with them. In addition, learners are provided with alternative activities that do not require computers if computer resources are not available for classroom use.

Session 1: Orientation

Much of the *Orientation* session is spent getting to know the course and meeting colleagues online. Learners also read about the approach used implemented in this course to learn and teach algebra.

Learners will:

Read:

- *The Landscape of Learning* - A discussion on the underlying principles behind the learning in this course and what they should expect.
- *The Landscape of Algebra* – A discussion about the structure of functions that underlies algebra, and how this structure can help students make connections
- *Nouns, Verbs and Mathematics*- A discussion about the importance of being able to view mathematical expressions in more than one way.

Complete activities and assignments:

- Create personal homepage providing a biographical sketch.

Write in online journal:

- Reflect on insights and ideas related to the three discussion readings: *The Landscape of Learning, The Landscape of Algebra* and *Nouns, Verbs and Mathematics*.

Participate in online discussion:

- Post messages in the Discussion Board, Session 1, *Getting to Know You.*
- Share thoughts on "The Landscape of Learning" and Nouns and Verbs and Mathematics in the Discussion Board, Session 1, *Landscape of Learning and Nouns, Verbs, and Mathematics.*

Session 2: Math Focus

This session is all about perspective. It begins with a glimpse of typical ways that textbooks approach quadratic functions and equations. Then learners solve a puzzle, searching for number patterns which they express symbolically. As they review different approaches to the puzzle, they will gain appreciation for the way various perspectives lead to the underlying mathematics, and see how quadratic functions arise from physical and geometric situations.

In addition to perspectives on the mathematics itself, learners adopt the viewpoints of learner and problem solver. The focus is on their experience with math: observe their own problem-solving processes, and thus gain insight into their learning (and ultimately, how it affects the way they think about how their students learn).

Learners will:

Read:

- *Snapshots from the Curriculum*: Looking at textbook examples through the lens of quadratic functions
- *Observing Your Processes*: Guidelines on observing personal problem solving.
- *Problem Solving Approaches, An e*xploration of some approaches that the *Got a Plan*? problem generates (approaches learners may not have used the first time)

Write in journal (not required):

- Reflect on approaches used to solve the *Diving In* problem and record findings such as:
  - What did they find so far and the trials that got them there
  - The tools and skills used most naturally
  - The issues that emerged in their minds as they worked on the task
- Write observations, and answers to the questions that follow each of the four challenges in the *Different Solutions to the Same Problem* reading such as:
  - What issues did Challenges 1–4 raise?
  - What value did they find in engaging different approaches to the same problem? How does mapping symbols *back* to the situation they represent support learning about symbols in algebra?

Participate in an online discussion:

- Post findings and observations in the Discussion Board, Session 2, *Diving In: The Toothpick Puzzle*. Then begin the discussion with their colleagues by reading their findings and responding to one or two of them.
- Share experiences and responses in the Discussion Board, Session 2, *Different Solutions to the Same Puzzle*.

Complete activities and assignments:

- *Diving In: Toothpick Problem* - Build a series of squares with toothpicks; look for patterns to predict the number of toothpicks in any size figure; write a rule that describes pattern.
- *Different Solutions to the Same Problem* - Explore and describe different solutions to the Toothpick Puzzle.

Session 3: Student Thinking

This session focuses principally on deciphering the mystery of student thinking. It interweaves two threads: observing students at their mathematical work, and extending learners ideas through more problem solving.

Videos show students working on a version of the *Diving In* activity that learners worked on during the previous session. Learners practice listening and observing students as they find and articulate patterns and rules, and then explore ways to support student understanding. They go behind the scenes to hear the goals and thoughts of the teacher, Barbara Shreve, and the comments of mathematics specialist, Deborah Schifter. They then delve a little more into the toothpick puzzle. Finally, they acquaint themselves with the Quadratic Transformer, software that lets links symbolic and graphic representations.

Learners will:

Read:
- *Meet the Students* –Background information about the students in the videos.
- *Don't Miss* - Items from student video
- *Don't Miss* - Items from Teacher and Specialist Commentary

View videos:
- Observe three boys figuring out a pattern and trying to express it algebraically in the video sequence *Looking for Patterns*, *How Do We Write the Rule?* and *Is It Really a Rule?*
- Watch Teacher and Specialist Commentary videos: Listen to the thoughts of the teacher, Barbara Shreve, and also consider the comments of a mathematics educator, Deborah Schifter, Ph.D. as they discuss the underlying mathematical themes in the students' work.

Write in Journal (not required):
- Reflect on what learners considered interesting or significant in the video, including any responses to the "Don't Miss" issues. What are the highlights of their observations?
- Reflect on what learners considered interesting or significant in the commentary by Barbara Shreve and Dr. Schifter.
- Images of the three toothpick puzzles from *Digging Deeper* and the function rules learners wrote for each:
  - Compare the three symbolic rules. Identify their differences and their similarities.
  - How do the differences identified reflect the differences among the images?
  - How do the similarities identified reflect the similarities among the images?

Participate in an online discussion:
- Post summaries of reflections from observing students on the Discussion Board under Session 3, *Observing Student Thinking.*

- Post responses in the Discussion Board, Session 3, *Teacher and Specialist Commentary* to the following:
  - Both teacher and specialist commented on group work as a means to learning. What evidence of that is prevalent in the videos of the students? How do they (learners) use group work? What opportunities or challenges do learning through group work present to them and their students?
  - Both teacher and specialist mention classroom practices needed to establish an environment where students work together, not just in parallel. What practices do learners use (or want to use) to encourage true collaboration among students?
  - What features do learners incorporate in a mathematical activity when they are preparing it for in-class group work?

Complete activities and assignments

- *Digging Deeper* - Find function rules that describe two variations on the toothpick pattern, and then compare their symbolic forms. (Later in the course, learners will link symbolic forms, tables, graphs, and the original figures.)
- *Quadratic Transformer Warm*-up - Activity to acquaint learners with software before using it.

Session 4: Your Classroom

Learners start by tying ideas from the Toothpick Puzzle to general concepts that underlie quadratic functions. Using the Quadratic Transformer, they connect physical components to tabular, symbolic, and graphic representations. As they work with a single function, linking the characteristics of one representation to those of another, they will discover the versatility of numeric, symbolic, graphic, and verbal representations.

In addition, they deepen their experience of quadratic functions and the many situations they can model. The ultimate goal of this session is to enable learners to integrate the various ideas from the course, and consider how they apply to their own situation and curriculum.

Learners will:

Watch Video:

- Watch as mathematics specialist, Dr. James Kaput, highlights the concepts that can be drawn from the cell phone and other models.

Write in Journal (not required):

- Reflect in journal how did the Piecewise Linear Grapher make the concepts in this activity more tangible? How might it enhance their students' understanding of the characteristics of linear functions? How might they use or adapt this activity for their classrooms?
- Consider, in more depth, the types of models they would like to use in developing their problems for their students, and the concepts that might arise through those models.

Complete activities and assignments:

- *Symbols and Graphs* - Connect symbols, tables, and graphs back to their original referents—the toothpick patterns.
- *Modeling Situations* - Make the connections of characteristics of quadratic functions to real world situations that they model

- *Adapt a Problem* - Look for real world situations or problems that can be modeled by quadratic functions from their curriculum and adapt one of these problems to provide a richer context for learning quadratic functions

Participate in an online discussion:

- Review responses to the questions in this activity and in *Digging Deeper* (from last session). Post experiences in the Discussion Board, Session 4, *Symbols and Graphs.*
  - How looking deeper into one puzzle facilitates the work on new puzzles
  - How they developed versatility in expressing the same idea in different ways through different forms
  - How that versatility might pay off when working with students
- Post solutions to *Modeling Situations* to the Discussion Board, Session 4, *Modeling Situations*. Include descriptions of the characteristics that they used to make the identifications and the other characteristics that *could* be used. Read through colleagues' postings
- Post in the Discussion Board, Session 4, their *Adapt a Problem*.problem and description of what they took into account as they adapted it. Read their colleagues' postings

Session 5: Your Plan

In this session learners will look back over the landscape of ideas they explored during this course. They will review their thoughts and records to consolidate their learning experience to create a project that will integrate the algebraic concepts developed throughout the course into their teaching practice. Learners will also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- A review of major topics addressed in this course

Complete activities and assignments:

- Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy learner want to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills

Participate in an online discussion:

- Post in the Discussion Board, Session 5, *Gallery of Plans-learners' completed plans*

Write in online journal (not required):

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Schedule

This course is designed to be a 30 hour course conducted over 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System
For the best experience, use the newer operating systems: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser
Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.3.1 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course is evaluated on a letter grade basis, and may be available for graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require learners to share their thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.

- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criteria for each assignment. The facilitator will apply individual criteria as necessary (for instance, not all activities require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.