Seeing Math Course Syllabus

Title

Seeing Math™**:** Quadratic Equations

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

Learners taking this course should have:

- Good grounding in algebra and comfort with linear functions, slope, and graphing straight lines
- Understanding of the differences between linear and quadratic functions
- Comfort with traditional methods of solving quadratic equations
- Some familiarity with the various forms that a quadratic function may take: polynomial, $y = ax^2 + bx + c$, vertex $y = a(x-h)^2 + k$, and root $y = a(x-r_1)(x-r_2)$
- Familiarity with graphing parabolas and ability to graph multiple functions on the same axes
- Ability to identify significant points on a parabola and how these are reflected in symbolic expressions
- Grasp of the coefficients in symbolic expressions of quadratic functions, and how the coefficients are reflected in graphs
- Familiarity with the symmetry of the parabola and with transformations of the parabola

Course Description

Through the course, Transformations of Quadratic Equations, learners and ultimately their students, can move beyond tried-and-true manipulations to examine the breadth of information available from quadratic equations. Learners look at the big picture: what the results tell, how to interpret them within the context of a problem, and how to find related information. Learners manipulate the three symbolic forms of a quadratic function in order to inspect and predict shape, orientation, and location and connect graphical and symbolic representations.

The learning in the course complements the typical textbook approaches to solving quadratic equations. In the end, it enables learners to answer the question "How can solving quadratic equations help us analyze quadratic functions, and what does the resulting information tell us, graphically, symbolically, or in a real world context?"

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Learners use quadratic equations to identify the defining characteristics of a quadratic function.

Learners will be able to:

- Distinguish how the characteristics are represented symbolically and graphically

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- Recognize patterns and relationships among certain characteristics of the function (for example, deducing the *y*-intercept by noticing patterns in the roots.)
- Describe and justify the meaning of the vertex, *y*-intercept, and *x*-intercept(s) by analyzing the behavior of a quadratic function at and around those points

Learners use different symbolic forms of a quadratic function to find important information.

Learners will be able to:

- Use the relationships between the three symbolic forms to gather information about the function at critical points
- Solve quadratic equations by representing and analyzing the function graphically and symbolically
- Connect traditional procedures for solving equations (for example, the quadratic formula) to symbolic and graphical representations of the function

Learners link personal understanding of quadratic equations to the curriculum and to student understanding.

Learners will be able to:

- Gain skill in interpreting student thinking about quadratic functions
- Interpret solutions to quadratic equations and identify information about critical points of the functions in the context of real world situations
- Adapt curriculum materials to bring the learning from this course into your classroom

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal; the discussions in the discussion forum are designed to allow learners to glean information from other learners' experiences.

This five-week course is taken entirely over the Internet. Learners should expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course, learners focus on creating and completing a final project.

Learners also come away with a tangible benefit—interactive software and activities to use with students. These tools are used within the course, such that learners are thoroughly familiar with them. In addition, learners are provided with alternative activities that do not require computers if computer resources are not available for classroom use.

Week 1: Orientation

Much of the *Orientation* week is spent getting to know the courseware and meeting colleagues online. Learners also read about our approach to learning and teaching algebra, look at NCTM and state standards, and begin their course journal.

Learners will:

Read:

- *The Landscape of Learning* – A discussion on the underlying principles behind the learning in this course and what they should expect.
- *The Landscape of Algebra* – A discussion of how the pieces fit together

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- *Nouns, Verbs, and Mathematics* – A discussion on both a practical and imaginative way to understand functions.
- National Council of Teachers of Mathematics *Algebra Standards*.
- Local state standards

Write in journal:

- Reflect on insights and ideas related to national and local standards on quadratic functions and transformations of quadratic functions? How does their curriculum currently address them? Do they see any that present particular challenges or any that they would like to understand or implement better?

Participate in online discussions:

- Post messages in the Discussion Board, Week 1, *Introductions* forum by sharing "get to know you messages. "Read what colleagues have to say and respond to at least two other posts.
- Post comments in the Discussion Board, Week 1, *Nouns, Verbs, and Landscapes.* Read what colleagues have to say and respond to at least two other posts. *Focus is on the following:*
  - Was this way of viewing algebra, as objects and processes, new? Did they find it a useful framework for talking about algebra (or other mathematical concepts)?
  - Can they think of situations (either when they were in school, or now as a teacher) in which this way of viewing and talking about algebra might be helpful?
  - Consider a particular case, such as $(x-2)^2+3$. When does their focus on $(x-2)^2+3$ shift between procedure and object? What other cases, from their teaching, do they naturally shift between these two viewpoints? Do their students make the switch with them?
  - How does the landscape of functions fit into the way they think about algebra? What ways does it fits in their present curriculum?
  - Did they find themselves using a similar orientation when they look at a problem?

<u>Week 2: Math Focus</u>

In this session learners explore the connection between the graphical representation of a quadratic function and its three symbolic forms. This practice will help them analyze quadratic functions from multiple perspectives. They start with a look at common textbook treatments for solving quadratic equations. This reading provides a basis of comparison for the approaches they experience in the course as well as some insight into how and why solving quadratic equations is addressed in algebra. Then, after a guided introduction to the Quadratic Transformer interactive, they examine relationships among the different forms of a quadratic function. Learners look at *families* of quadratic functions—functions that have common characteristics—and use the roots along with other characteristics to garner further information about the functions. The goal will be to develop symbol sense as you connect symbolic representations to their graphical counterparts. Throughout the week, they focus on their own problem-solving processes as a way to gain insight into how students might solve the same problems, and share their thinking and solutions with their online colleagues.

Learners will:

Read:

- *Snapshots from the Curriculum*: Teaching Transformations: a presentation of common ideas from textbooks and ways to bridge any gaps.
- *Observing Your Processes*: Guidelines on observing personal problem solving.

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Complete activities and assignments:

- Warm Up: Get to know the Quadratic Transformer software.
- Diving In, Analyzing Quadratic Functions – Look at symbols and graphs, forms and families

Write in journal (not required):

- Reflect on approaches used in the *Diving In*: *Analyzing Quadratic Functions*
  - Reflect: How did they make these parabolas? (Did they start with the graphs and then examine the symbolic representation, or vice versa?)
  - Did they find themselves leaning on a particular method, or representation, as the most comfortable one?

Participate in online discussions:

- Post responses to Questions 1 and 2, in the Discussion Board, Week 2, *Diving in: Analyzing Quadratic Functions*. Read your colleagues posting and respond to at least two of them.
  1. Choose Challenge C, D, or E and describe how they found their answers.
     - How did they make their parabolas?
     - What strategies did you use to make your predictions or answer questions?
     - Which approaches were the natural first choices?
     - Describe their approaches so that colleagues understand.
  2. Respond to one or two of the questions below.
     - What kinds of perspectives did exploring *families* of parabolas help them gain about the characteristics of quadratic functions?
     - Did exploring these characteristics using multiple representations offer new insights about the solutions to quadratic equations?
     - How did the software influence their approach to the tasks?
     - Are any ideas or concepts still unclear?

<u>Week 3: Student Thinking</u>

In this session, learners focus on students as they work on some of the same activities. As learners listen to math conversations, they gain insight into student thinking so that they will be better able to listen closely to their own students and understand their approaches. After seeing the student videos, they hear additional insight from our specialist commentator, Henri Picciotto. Finally, learners deepen their examination of the three symbolic forms a quadratic function can take.

Learners will:

Read:

- *Meet the Students* – Background information about the students in the videos.
- *What's Happening and Don't Miss* comments regarding the student videos
- *What's Happening and Don't Miss* comments regarding the Specialist Commentary

View videos:

- Observing Student Thinking – Students tackling on quadratic equations.
- Mathematics education specialist, Henri Picciotto, describing the mathematics in the tasks, how the students approached the problems, and the implications for teaching and learning quadratic functions and equations.

Complete activities and assignments:

- Two Parabola Challenge – Some interesting challenge problems that provide the opportunity to apply what has been learned about quadratic functions.

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Write in journal (not required):

- Reflect on the students' processes as observed in the videos: Record notes of what was considered interesting or significant in the videos, and thoughts on the "Don't Miss" items. Choose two or three issues or questions that stood out and summarize them.
    - What discoveries do the students in the videos make?
    - What stumbling blocks do they encounter?
    - Some people view technology as a catalyst for student thinking; others see it as a crutch. Think about the conversations observed in the student videos—how can the Quadratic Transformer (and associated activities) advance understanding of the relationships between symbolic and graphical representations, and of techniques for analyzing quadratic functions? How might it obscure their thinking? Support ideas with evidence from the videos.
- Reflect on interesting or significant ideas from Picciotto's commentaries:
    - How did Picciotto's ideas about student thinking, or about the activity, resonate with their own experiences?
    - What can they bring to their classrooms that will help them move to more careful and accurate listening to students?
- Reflect on notes from *Diving In: Analyzing Quadratic Functions*. Review those notes as well as the work completed in the Focus on Symbols activity. In preparation for the discussion, review the original goals of this activity and focus thoughts on the following questions:
    - How did the exercises help them describe and justify the meaning of *y-intercept, root,* and *vertex*, both graphically and symbolically?
    - In *Focus on Symbols* they may have found some of the symbolic forms familiar and comfortable, but others less so. What interesting discoveries did they make as you worked on *Focus on Symbols*?
    - In *Diving In* and its follow-up, *Focus on Symbols*, the approaches of manipulating the symbols or directly comparing the graphs and symbols helped them gather information about the function in different ways. Based on these experiences with this session's activities, what are some of the benefits and limitations of these two approaches?
    - One goal of this activity was to develop symbol sense by connecting symbols to their graphical representations. However, generalizing with symbols can be challenging, even frustrating. If this activity is implemented with students, what challenges do they anticipate they might face? What strategies might be used to help students tackle these concepts and move out of their comfort zones, while allowing them to achieve a degree of confidence and success?

Participate in online discussions:

- Review notes on what was considered significant in both Picciotto's commentaries and in *Observing Student Thinking* videos. Post summary of thoughts to the Discussion Board, Week 3, *Student Thinking and Specialist Commentary*. Read what colleagues have to say and respond to at least two other learners.
- Share thoughts and ideas emanating from the Focus on Symbols activity. Post comments in Discussion Board, Week 3, *Focus on Symbols.* Read your colleagues' postings, and respond to at least two of them.

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Week 4: Your Classroom

In this session, learners apply the new perspectives they have gained in this course to their own classroom situations. First, they carry out an activity to experience an alternative derivation of the quadratic formula—one that connects the graphical and symbolic representations they have done in previous activities. Then they hear further comments by specialist Henri Picciotto. Finally, learners choose a problem from their own curriculum and adapt it to integrate the themes explored in the course so far.

Learners will:

Read:

- *A Graphical/Symbolic Approach to Solving Quadratic Equations* - Derive the quadratic formula using an alternative approach that combines symbolic and graphical representations.

Watch videos:

- Specialist Commentary: Part 2 – Mathematics education specialist, Henri Picciotto on the role of formulas and vocabulary in the math classroom.

Complete activities and assignments:

- *Adapt a Problem*
  - Explore curriculum and find problems that can be modified to integrate the concepts studied during this course.
  - Adapt one of these problems to increase learning beyond what the original problem provided.
  - Describe thinking and adaptation.
  - Share adapted problem in the Discussion Board, and view colleague's problems.

Write in journal (not required):

- Reflect on interesting or significant points in Picciotto's commentaries regarding:
  Formula sense:
  - When the use of formulas in mathematics support student understanding?
  - When are formulas dissociated from meaning?
  - When is memorization detrimental to learning?

  A time and place for learning:
  - What does Picciotto and French educators mean by "institutionalizing the learning" in order to promote a universal mathematical language.

Participate in online discussions:

- Post discoveries and responses to the following questions in the Discussion Board, Week 4, *A Graphical/Symbolic Approach.* Read colleagues' postings and respond to at least two other posts. Consider the following in the discussion:
  - How does different way to find the *x*-intercepts of a quadratic equation compare to the other method learned or currently taught?
  - How might it be used to advance student thinking?
  - How might they break it down or add to it so students can understand it?
- In the Discussion Board, Week 4, *Adapt a Problem,* post the problem, and description of what was taken into account in its adaptation. Include the original problem, the adapted problem, and your brief description. You might also want to include student handouts if you created them. Read colleagues' postings and reply to at least two of them.

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Week 5: Your Plan

In this session learners look back over the landscape of ideas they explored during this course. They review their thoughts and records to consolidate their learning experiences and build upon the work completed over the past weeks. Learners create a final project designed to integrate the mathematical concepts developed throughout the course into their instructional program. Learners also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- A review of the major topics addressed in this course

Complete activities and assignments:

- Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy that the learner wants to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills.
- Post in the Discussion Board, Week 5, *Gallery of Plans-learners' completed plans.*

Participate in an online discussion:

- Post Aha!, Oops! and/or Whew! comments on the Discussion Board Week 5, *Aha! Oops! Whew!* Say goodbye to fellow learners, share their memories of this course experience, and plan for continued professional contact. Read what colleagues have to say and respond to at least two other posts.

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Write in journal (not required):

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Schedule

This course is scheduled to take approximately 30 hours to complete. The number of hours identified for each course reflects time spent online, but does not reflect the total time spent completing offline coursework and assignments. All learners are different and learners will likely spend double the indicated number of hours completing all coursework depending on learning styles and work habits.

Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

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Operating System
For the best experience, use the newer operating systems: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser
Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players
One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.5 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course is evaluated on a letter grade basis, and may be available for graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

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Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require learners to share their thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.

The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criterion for each assignment. The facilitator will apply individual criteria as necessary (for instance, not all activities require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.

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Action Plan Rubric

Criteria	**Does not meet expectations**	**Meets expectations**	**Exceeds expectations**
Completion of Assignment and Timeliness	Assignment is not completed and/or submitted on time.	Assignment is completed and submitted on time.	N/A
Description of the instructional strategy	A specific action or instructional strategy is not included and/or reason(s) why this strategy was selected are not included.	A specific action or instructional strategy is included, along with the reason(s) why this strategy was selected.	A specific action or instructional strategy and reason(s) why this strategy was selected are included. In addition, specific examples from experiences in the course or from personal reflections that were instrumental in the selection of the stated strategy are included.
Goals	The goals for the teacher and/or the students based on the instructional strategy selected are not included.	The goals for both teacher and student based on the instructional strategy selected are included.	The goals for both teacher and student based on the instructional strategy selected are included. Goals are SMART (see note below).
Objectives	The specific performance objectives teacher will do to accomplish the stated goals for the selected strategy are absent.	The specific performance objectives the teacher will do to accomplish the stated goals for the selected strategy are clearly described.	The specific performance objectives the teacher will do to accomplish the stated goals for the selected strategy are clearly described. A rationale for each of the stated objectives is provided.
Potential activities	The plan does not delineate and/or describe the specific activities/tasks to be undertaken to meet the objectives.	The plan delineates and briefly describes the specific activities/tasks to be undertaken to meet the objectives.	The plan delineates and briefly describes the specific activities/tasks to be undertaken to meet the objectives. The activities indicate attention to ensuring the needs of all students are met.

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Relevance	How the scope of the plan is relevant to the study of algebra, or to the students, or the school or district, and to the quality of educational practice is not stated.	How the scope of the plan is relevant to the study of algebra, or to the students, the school or district goals, and to the quality of educational practice is stated.	How the scope of the plan is relevant to the study of algebra, to the students, the school or district goals, and to the quality of educational practice is stated.
Evaluation	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are not described.	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are described.	The criteria to be used in determining the success of the objectives, including when and how the plan will be adjusted, if needed, are described. A brief student evaluation survey is also included.
Portfolio	Pieces of evidence from the proposed activities that will be collected for each objective are not identified.	Pieces of evidence from the proposed activities that will be collected to support each objective are identified.	Pieces of evidence from the proposed activities that will be collected to support each objective are identified. Also included is a rationale statement for selecting at least four pieces of evidence.

Note: SMART refers to Specific, Measurable, Attainable, Relevant, Time-bound.

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Lesson Plan Rubric

Criteria	Does not meet expectations	Meets expectations	Exceeds expectations
Completion of Assignment and Timeliness	Assignment is not completed and/or submitted on time.	Assignment is completed and submitted on time.	N/A
Goals	Goals are not provided.	Goals clearly state the purpose of the lesson. There is evidence of the alignment of the goals, objectives, learning activities, and assessment methods described in the lesson plan.	N/A
Objectives	Objectives are not provided.	Objectives are performance-based, state what students should know and be able to do as a result of learning instruction. Objectives are appropriate for the grade level and students identified. There is clear evidence of the alignment between the goals, objectives, learning activities, and assessment methods described in the lesson plan.	N/A
Standards Addressed	National and state content and technology standards are not addressed.	National and state content and technology standards are listed.	N/A
Prerequisites	Prerequisite knowledge and skills are not provided, or prerequisites are vague, or prerequisites are not appropriate.	Appropriate prerequisite knowledge and skills needed by students are provided.	Appropriate prerequisite knowledge and skills needed by students are provided. An explanation of their importance to the learning is provided.

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Materials	Materials and resources are not listed, or only a partial list is provided.	A complete list of materials, resources, and detailed descriptions of any special considerations and/or advanced preparations are provided.	A complete list of materials, resources, and detailed descriptions of any special considerations and/or advanced preparations are provided. A list of additional/alternative materials and resources is also provided.
Lesson Overview	The lesson overview is not provided or lesson overview is incomplete, vague or unclear.	The lesson overview provides a brief statement that summarizes key aspects of the lesson.	N/A
Teaching Strategy	Lesson design does not document clearly the teaching strategy that needs to be implemented, whether it is direct instruction or learner-centered instruction.	Lesson plan documents clearly the appropriate teaching strategy needed for the lesson, whether it is direct instruction or learner-centered instruction.	Lesson plan documents clearly the appropriate teaching strategy needed for the lesson, whether it is direct instruction or learner-centered instruction. A rationale for the selection based on course readings and best practices is provided.
Lesson Procedures		Lesson procedures provide a detailed, step-by-step description of the lesson. They include: Introduction — how students will be introduced to the goals and what is expected of them Main activity — how the teacher will facilitate the learning experience Conclusion — how the teacher will bring closure for students and provide feedback Extension — how the lesson will be extended.	Lesson procedures provide a detailed, step-by-step description of the lesson. They include: Introduction — how students will be introduced to the goals and what is expected of them Main activity — how the teacher will facilitate the learning experience Conclusion — how the teacher will bring closure for students and provide feedback Extension — how the lesson will be extended. Procedures include strategies for differentiated instruction.

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Assessment	Assessment is not provided, is incomplete, and/or vague. There is not a clear relationship between the assessment and the skills taught during the lesson.	Assessment(s) to be used to evaluate students' learning is (are) provided. There is a clear relationship between the assessment(s), the content, and the skills taught during the lesson.	Assessment(s) to be used to evaluate students' learning is (are) provided. There is a clear relationship between the assessment(s), the content, and the skills taught during the lesson. Assessment(s) incorporate(s) a consideration for diverse student needs. A rationale for the selection of the assessment technique(s) based on course readings and best practices is provided.

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